Rare Events in Turbulence Might not be So Rare

Susan Kurien, T-7

We have shown, using a compilation of data from experiments and direct numerical simulations, that intense, intermittent events are not merely associated with the rare events in the flow but are in fact present in the high frequency events. Thus our results motivate the theoretical study of turbulence statistics in the limit of zeroth order moments, unbiased by so-called rare events which might be flow-dependent. This report is a summary of work published in [1].

In 1941, the Russian mathematician Andrei N. Kolmogorov deduced scaling laws for the velocity statistics of turbulent flows. At the core of the Kolmogorov theory lies the hypothesis that the scales in fully developed turbulence are statistically self-similar. The theory predicts that the n-th order moment of the velocity difference across scales of size r should scale as $r^{n/3}$. Experimental data has shown that for $n > 3$ the scaling exponents are $\zeta_n < n/3$. It is now believed that the turbulent scales are not self-similar but intermittent and the scaling exponents of moments of velocity increments are anomalous, that is, the departures from Kolmogorov's self-similar scaling increase nonlinearly with the increasing order of the moment. Since high-order moments sample the tails of a probability distribution function, it is also believed that the intermittency in turbulence is associated with rare events. We have shown, using a compilation of data from experiments and direct numerical simulations, that intermittency is not merely associated with the rare events in the flow but is in fact present in the high frequency events. Thus our results motivate the theoretical study of anomalous scaling in the limit of zeroth-order moments, unbiased by so-called rare events. The moments of the velocity difference across spatial scales are known as structure functions and are defined by

$$S_n(r) = \langle |\,\delta u(r)\,|^n \rangle = \int_0^\infty |\,\delta u(r)\,|^n P(\delta u(r))d(\delta u(r))$$

where $\delta u(r) = (u(x+r) - u(x)).\ \hat{r}$ is the longitudinal velocity increment across scales of size r. The $\langle . \rangle$ denotes an ensemble average which in practice is implemented as an average over the space-time domain. $P(\delta u(r))$ is the probability density distribution. In the probabilistic sense, it is clear from the above equation that the large n moments are associated with tail of P, or the low probability events. Structure functions are useful measures of the statistical properties of turbulent flows as a function of scale. In particular, the second-order moment of velocity difference across scales of size r is a measurement of the energy contained in those scales. Kolmogorov derived an exact law from the energy balance equation governing the third-order structure function, yielding $S_3 \sim r$ for homogeneous and isotropic turbulence. The self-similarity assumption then yields for statistically isotropic turbulence $S_n(r) \sim r^{n/3}$. Empirical measurement of self-similar scaling for $n = 2$ and 3 have been consistent with the Kolmogorov theory. However, as higher-order moments began to be reliably measured, it became clear that there is a pronounced departure from the self-similar scaling laws (so-called *anomalous* scaling).

We analyzed three sets of data in this work. Experimental measurements of velocity fluctuations in the atmospheric boundary layer, resolved direct numerical simulations (DNS) of the forced Navier-Stokes equation in a periodic cube of 512 grid-point to a side, and another computed in a cube of 1024 grid points to a side. The figure shows the relative departure from the Kolmogorov prediction, of scaling exponents of moments of order

-0.8 ≤ n ≤ 10. Most current theoretical efforts are focused on the intermittency for integer $n > 3$. The empirical results presented here show that there is significant anomalous scaling for moments of order $n < 3$ provide motivation to seek a theoretical explanation for intermittency in the high frequency events, and correspondingly for anomalous scaling in the low-order moments.

For more information contact Susan Kurien at skurien@lanl.gov.

[1] S. Chen, et al., *J. Fluid Mech.* **533**, 183–192 (2005).
[2] C. Meneveau and K.R. Sreenivasan, *Phys. Rev. Lett.* **59**, 1424–1427 (1987).

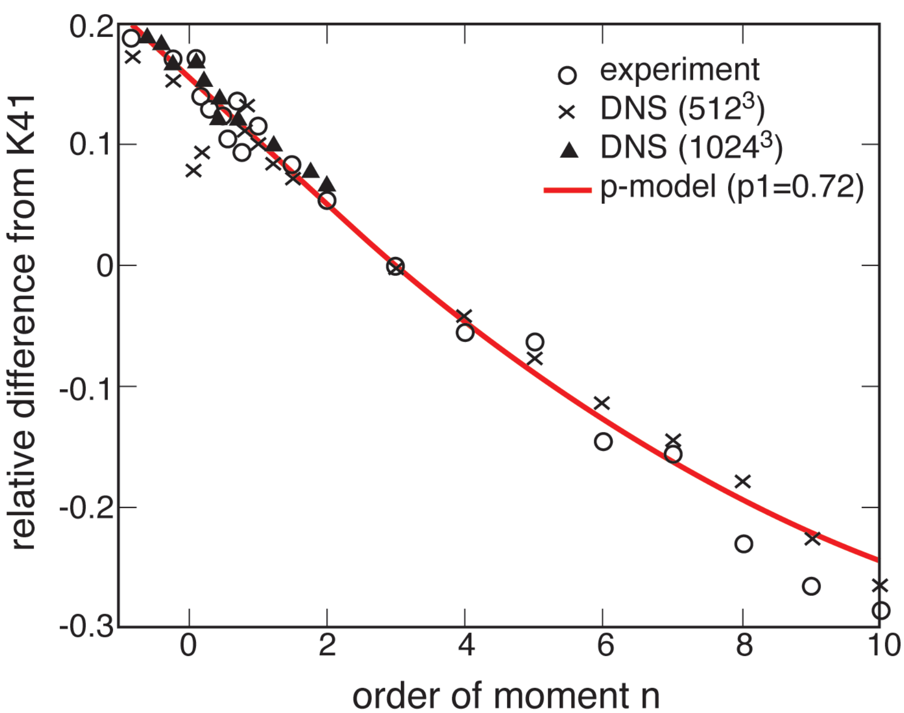

Fig. 1.
The relative departure from the Kolmogorov1941 self-similarity theory for moments ranging from -0.8 ≤ n ≤ 10 as computed from various sources of data. The solid line is the comparison to the so-called p-model [2] derived by assuming multifractality of the scales. The important feature to note is that for n < 2 the absolute relative departure from Kolmogorov scaling increases at the same rate as for n > 2.